Physics II
ISI B.Math
Mid Semestral Exam : February 23, 2011

Total Marks: 100
Answer all questions. All questions carry equal marks

1. For each of the following statements indicate whether it is true or false and give a brief explanation. A few lines should suffice. (20)

a) If A and B are two equilibrium states of a system the change ΔS_{AB} for the entropy of the system when it changes from A to B will be greater for an irreversible process than that for a reversible process.
b)A Carnot engine whose thermal efficiency is very high is particularly well suited for a refrigerator, when run in the reverse direction.
c)A Carnot engine is operated using a gas that obeys Van der Waals equation of state. The change of Gibbs free energy over a cycle of this engine is equal to zero.
d)On a PV plot, one can always find a pair of adiabatic curves that intersect each other.
e)$dF = (x^2 - y^2)dx + xdy$ is an exact differential.
f)The work done in an adiabatic process depends on the path
g) If He and O_2 are assumed to be ideal gases, they will have identical molar heat capacities at constant pressure.
h)An ideal gas is enclosed in a box with adiabatic walls. One of the walls has a valve, which when opened, allows the gas to freely expand into a larger box with adiabatic walls. $< v^2 >$ of the molecules of the gas decreases after the valve is opened.
i) A Carnot cycle, represented on a $S-T$ diagram, will always take the form of a square, irrespective of the working substance.
j)One litre of nitrogen gas at 0° C is enclosed in a cylinder fitted with a piston. It is adiabatically compressed until the temperature increases to 100°C. It is then allowed to expand back to 1 litre isothermally in contact with a reservoir at 100°C. The entropy change of the universe in this process is zero.

2. (a) Starting from the first law of thermodynamics and the definitions of

C_p and C_v, show that (7)

$$C_p - C_v = \left[p + \left(\frac{\partial U}{\partial V} \right)_P \right] \left(\frac{\partial V}{\partial T} \right)_P$$

(b) Use the fact that entropy is a state function to show that (7)

$$\left(\frac{\partial U}{\partial V} \right)_p = T \left(\frac{\partial p}{\partial T} \right)_V - p$$

Use the results in (a) and (b) to calculate $C_p - C_v$ for a van-der Waals gas $(p + \frac{a}{V^2}) \times (V - b) = RT$. Show that you recover the ideal gas result in the limit $V \to \infty$ at constant p.(6).

3. In the big bang theory of the universe, the radiation energy initially confined in a small region adiabatically expands in a spherically symmetric manner. The radiation cools down as it expands. The internal energy density of the radiation or photon gas is given by $u = \sigma T^4$ where σ is a constant and the radiation pressure is given by $p = \frac{1}{3}u$.
(a)Derive a relationship between the radius R of the spherical volume of radiation and the temperature T, based purely on thermodynamic considerations (7)
(b) Find the total entropy of a photon gas as a function of its temperature T and volume V. (8)
(c) If a Carnot engine were made to operate with the photon gas as a working material, sketch the corresponding cycle on a P-V diagram.(5)

4. An adiabatic cylinder, closed at both ends, is fitted with a frictionless adiabatic piston that divides the cylinder into two parts. Initially the pressure, volume and temperature are the same on both sides of the piston (P_0, V_0 and T_0). The gas is ideal with C_V independent of temperature and $\gamma = 1.5$. By means of a heating coil in the gas on the left side, heat is slowly supplied to the gas on the left until the pressure reaches $27P_0/8$. In terms of nR, V_0 and T_0: (20)
(a) What is the final volume on the right side ?
(b) What is the final temperature on the right side?
(c)What is the final temperature on the left side ?
(d) How much heat must be supplied to the gas on the left side (ignore the

coil)?
(e) How much work is done on the gas on the right side ?
(f) What is the entropy change of the gas on the left side?
(g) What is the entropy change of the gas on the right side?
(h) What is the entropy change of the universe?

5. a) A thermally insulated box of volume $2V$ is partitioned into two boxes of equal volume. One half contains an ideal gas at temperature T and the other half is vacuum. The partition is removed and the gas is allowed to expand into the entire box. Show that this is an irreversible process by showing that the entropy of the universe increases in the process. (7)
b) An inventor claims that he has designed an engine that works in a cycle using two heat sources , one at 400K and the other at 300K and that the engine takes 400J of heat from the reservoir at 400K and produces an output of 300J of work in a cycle. Is his claim credible? Justify your answer.(5)
c) Show that for any PVT system, starting from $C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

Hence show that C_V for an ideal gas is a function of T only (5)
d) Represent a Carnot cycle with an ideal gas on a H-S diagram, i.e, enthalpy-entropy plot. (3)

Information you may (or may not) need

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$F = U - TS$$

$$G = U - TS + PV$$

$$H = U + PV$$